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CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

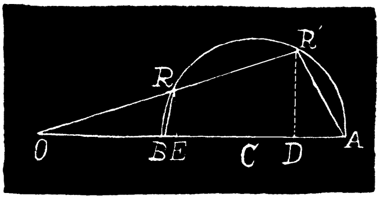
SOLUTIONS OF PROBLEMS.

31. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Through a point O on the produced diameter AB of a semicircle draw a secant ORR', so that the quadrilateral $ABRR'$ inscribed in the semicircle shall be a maximum. Prove that in this case, the projection of RR' on AB is equal in length to the radius of the circle. [*Williamson's Diff. Calculus*, 7th edition, p. 189, Ex. 25.]

I Solution by JOHN B. FAUGHT, A. B., Instructor in Mathematics, Indiana University, Bloomington, Indiana.

Let $OC=p$; then the equation of the circle is $\rho^2-2\rho p\cos\theta+p^2-r^2=0$. The roots of which are $\rho_1=p\cos\theta+\sqrt{(r^2-p^2\sin^2\theta)}=OR'$ and $\rho_2=p\cos\theta-\sqrt{(r^2-p^2\sin^2\theta}=OR$.



$\triangle OAR=OA.OR'\sin\theta=(p+r)[p\cos\theta+\sqrt{(r^2-p^2\sin^2\theta}]\sin\theta$.

$\triangle OBR=AB.OR\sin\theta=(p-r)[p\cos\theta-\sqrt{(r^2-p^2\sin^2\theta}]\sin\theta$.

$\therefore$ Qd. $ABRR'=2p[r\sin\theta\cos\theta+\sin\theta\sqrt{(r^2-p^2\sin^2\theta}]$.

$$\therefore \frac{d(\text{Qd})}{d\theta}=\frac{r(1-2\sin^2\theta)\sqrt{(r^2-p^2\sin^2\theta)}+\cos\theta(r^2-2p^2\sin^2\theta)}{\sqrt{(r^2-p^2\sin^2\theta)}}=0,$$ for a maximum or minimum.

Reducing we have $4p^2\sin^4\theta-4(p^2+r^2)\sin^2\theta+3r^2=0$.

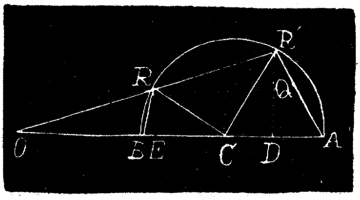
$$\therefore \sin\theta=\frac{\sqrt{(r^2+p^2)\pm\sqrt{(r^4-r^2p^2+p^4)}}}{p},$$ which must be a maximum from the nature of the problem.

$$ED=OR'\cos\theta-OR\cos\theta=(\rho_1-\rho_2)\cos\theta=2(\sqrt{r^2-p^2\sin^2\theta)}\cos\theta$$
$$=\sqrt{4(r^2-p^2\sin^2\theta)(1-\sin^2\theta}=\sqrt{4r^2-4(p^2+r^2)\sin^2\theta+4p^2\sin^4\theta}$$
$$=\sqrt{(4r^2-3r^2)}=r.$$

II. Solution by F. P. MATZ, M. Sc. Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland, and J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let $CB=r$, $CO=a$, $a/r=c$, $\angle COR=\theta$, $CR'O=\phi$; then $\angle ACR'=(\phi+\theta)$, $\angle OCR=(\phi-\theta)$, $OR'=r\sin(\phi+\theta)/\sin\theta$, $OR=r\sin(\phi-\theta)/\sin\theta$, and

$\sin\phi = c\sin\theta \ldots .(1)$. The area of the quadrilateral is $\mathbf{A} = \frac{1}{2}[AO + OR' - BO \times OR]\sin\theta, = ar[\cos\theta + \surd(1 - c^2\sin^2\theta)]\sin\theta \ldots .(2)$, which is to be a maximum.



Differentiating, etc., we have

$$\frac{d\mathbf{A}}{d\theta} = \frac{1-2\sin^2\theta}{1-2c^2\sin^2\theta} + \frac{\surd(1-\sin^2\theta)}{\surd(1-c^2\sin^2\theta)} = 0 \ldots .(3).$$

$\therefore\ 4c^2\cos^4\theta - 4(c^2-1)\cos^2\theta - 1 = 0 \ldots .(\alpha)$,

or $4c^2\sin^4\theta - 4(c^2+1)\sin^2\theta + 3 = 0 \ldots .(\beta)$.

From (α), $\cos^2\theta = \frac{1}{2}c^{-2}\{\surd[(c^2-1)^2 + c^2] + (c^2-1)\} \ldots .(4)$.

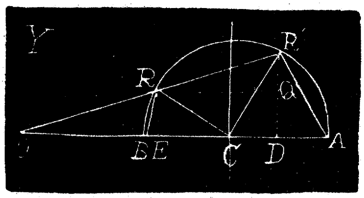
From (1), $\cos^2\phi = \frac{1}{2}\{\surd[(c^2-1)^2 + c^2] - (c^2-1)\} \ldots .(5)$.

The projection of RR' on AB is $\mathbf{P} = OR' - OR)\cos\theta, = 2r\cos\phi\cos\theta \ldots .(6)$.

Substituting the square roots of (4) and (5) in (6), etc., we have $\mathbf{P} = r$.

III. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let O be the origin, Ox, Oy the axes, C the centre of the circle, $OC = a$, $BC = AC = r$, $\tan ROB = m$.



Then equation to circle is $(x-a)^2 + y^2 = r^2$, and equation to line ORR' is $y = mx$.

By substitution we get

$$(x-a)^2 + m^2x^2 = r^2.$$

$$\therefore\ x = \frac{a \pm \surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2},$$

$$y = \frac{ma \pm m\surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2}.$$

Now area $OR'A$—area ORB=max. $\therefore\ \frac{1}{2}OA \times R'D - \frac{1}{2}OB \times RE$=max.

$$\therefore\ \tfrac{1}{2}(a+r)\left\{\frac{ma + m\surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2}\right\}$$

$$-\tfrac{1}{2}(a-r)\left\{\frac{ma - m\surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2}\right\} = \text{max}.$$

$$\therefore\ \frac{mar + ma\surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2} = \text{max}.$$

Reducing the first differential coefficient we get

$$r^2 + r^2m^2 - 2a^2m^2 = r(m^2-1)\surd(r^2 + r^2m^2 - a^2m^2.$$

Squaring and reducing we easily get $m^4r^2 + 2(2a^2 - r^2)m^2 = 3r^2$.

$\therefore\ m^2 = \frac{1}{r^2}[2\surd(a^4 - a^2r^2 + r^4) - 2a^2 + r^2]$, as the only admissible value.

The projection of RR' on $AB = ED = OD - OE = \dfrac{2\surd(r^2 + r^2m^2 - a^2m^2)}{1+m^2} = r$ for the above value of m^2.

When $a = r$, $m^2 = 1$. $\therefore\ \angle ROB = 45^\circ$.

Also solved by *Alfred Hume*, *C. E. White*, and *H. W. Draughon*.